An invisible medium for circularly polarized electromagnetic waves

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Abstract: We study the no reflection condition for a planar boundary between vacuum and an isotropic chiral medium. In general chiral media, elliptically polarized waves incident at a particular angle satisfy the no reflection condition. When the wave impedance and wavenumber of the chiral medium are equal to the corresponding parameters of vacuum, one of the circularly polarized waves is transmitted to the medium without reflection or refraction for all angles of incidence. We propose a circular polarizing beam splitter as a simple application of the no reflection effect.

OCIS codes: (260.2110) Electromagnetic optics; (160.1585) Chiral media; (160.3918) Metamaterials; (230.5440) Polarization-selective devices.

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1. Introduction

When an electromagnetic (EM) wave is incident on a boundary between two media, the incident wave is partially reflected. However, at a particular angle of incidence, the reflected wave vanishes. This phenomenon is known as the Brewster effect [1]. The Brewster no-reflection effect is utilized in many applications; for example, intracavity elements are placed at Brewster angles in order to suppress insertion losses.

The Brewster effect for transverse-magnetic (TM) waves (p waves) arises at an interface between two media whose permittivities are different from each other. The TM Brewster effect can be observed in naturally occurring dielectric media. The Brewster effect for transverse-electric (TE) waves (s waves) arises at an interface between two media whose permeabilities are different from each other [2–6]. Normally, it is difficult to observe the Brewster effect for TE waves in naturally occurring media because they do not respond to magnetic fields in high frequency regions, i.e., microwave, terahertz, and optical regions. However, magnetic media can be fabricated in high frequency regions by using metamaterials [7–11], and therefore, the TE Brewster effect can be observed. The effect has been experimentally confirmed in the microwave region [12] and also in the optical region [13].

In addition to permittivity and permeability, it is possible to control the chirality parameter and the non-reciprocity parameter by using metamaterials; therefore, it is important to explore the no reflection conditions for generalized media. Brewster conditions for various media have been studied by several researchers [2–6, 12–20]. Some researchers have derived the expres-

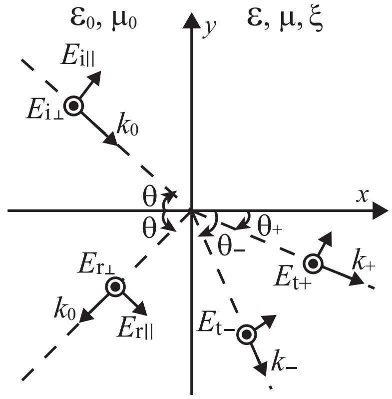

Fig. 1. Geometry of coordinate system. The incident, reflected, and transmitted waves are denoted by the subscripts i, r, and t, respectively. Region $x < 0$ represents vacuum, and region $x \geq 0$ represents the chiral medium.

sions of Brewster conditions for chiral and non-reciprocal media. For these media, the incident polarization for no reflection is not necessarily perpendicular nor parallel to the plane of incidence. The no reflection conditions cannot be found if the incident wave is assumed to be TM or TE waves. It has been pointed out that the Brewster condition must be revised such that the polarization of the reflected wave is independent of the incident polarization [16–18]. However, thus far, the explicit relations among the medium parameters for achieving non reflectivity have not been determined.

The objective of the present study is to explicitly derive the no reflection conditions for an (isotropic) chiral medium that responds to the electric and magnetic fields simultaneously. This class of medium, having relative permittivity $\varepsilon_r \neq 1$, relative permeability $\mu_r \neq 1$, and normalized chirality parameter $\xi_r \neq 0$, can be accessible with the current technology of metamaterials.

First, we confirm that the revised Brewster condition [16–18] reduces to the condition that the reflection (Jones) matrix has at least one vanishing eigenvalue. We show that the analysis can be largely simplified by using Pauli matrices [21].

It is found that in general chiral media, the no reflection condition is satisfied by elliptically polarized incident waves having a particular angle of incidence. This is merely a natural extension of the usual Brewster effect for achiral ($\xi_r = 0$) media. In addition, a qualitatively new mode of no reflection is found. When the wave impedance and wavenumber of the chiral medium, determined by ε_r, μ_r, and ξ_r, are equal to the vacuum values Z_0 and k_0, respectively, one of the circularly polarized waves is transmitted to the medium without either reflection or refraction. The no reflection condition is independent of the incident angle, i.e., the medium is totally transparent with respect to only that circular polarization. The other circularly polarized (CP) wave is refracted and reflected, or even totally reflected. The no reflection phenomenon can be physically understood as a destructive interference of electric and magnetic responses, due to the mixing through the chirality parameter.

A simple, straightforward application of the totally transparent medium for the circularly polarized wave is a circular polarizing beam splitter (CPBS). We analyze the CPBS by a finite-difference time-domain (FDTD) method [22]. The properties of this CPBS are almost ideal compared with earlier polarizing beam splitters [23–26].

2. No reflection condition for chiral media

As shown in Fig. 1, we suppose that an EM wave with an angular frequency ω is incident from vacuum (permittivity ε_0, permeability μ_0) on an isotropic chiral medium at an incident angle of θ. The constitutive equations for the chiral medium are given as [27–30]

$$\boldsymbol{D} = \varepsilon \boldsymbol{E} - \mathrm{i}\xi \boldsymbol{B}, \quad \boldsymbol{H} = \mu^{-1}\boldsymbol{B} - \mathrm{i}\xi \boldsymbol{E}, \tag{1}$$

where ε, μ, and ξ are the permittivity, permeability, and chirality parameter, respectively. We adopt the EB formulation for the constitutive equations, but it is also possible to rewrite them in terms of EH formulation [18]. Due to the translational invariance of the interface, Snell's equations $k_0 \sin\theta = k_+ \sin\theta_+ = k_- \sin\theta_-$ are satisfied. Here, $k_0 = \omega\sqrt{\varepsilon_0\mu_0}$ is the wavenumber in vacuum, $k_\pm = \omega(\sqrt{\varepsilon\mu + \mu^2\xi^2} \pm \mu\xi)$ are the wavenumbers for left circularly polarized (LCP) and right circularly polarized (RCP) waves in the chiral medium, and θ_+ (θ_-) is the refractive angle of the LCP (RCP) wave.

The relation between the electric field of the incident wave, $\boldsymbol{E}_\mathrm{i} = [E_{\mathrm{i}\perp}, E_{\mathrm{i}\|}]^T$ (T stands for transposition), and that of the reflected wave, $\boldsymbol{E}_\mathrm{r} = [E_{\mathrm{r}\perp}, E_{\mathrm{r}\|}]^T$, is written as [16]

$$\begin{aligned}
\boldsymbol{E}_\mathrm{r} &= \frac{1}{\Delta} M_\mathrm{R} \boldsymbol{E}_\mathrm{i}, \quad M_\mathrm{R} = c_\mathrm{u} I + c_2\sigma_2 + c_3\sigma_3,\\
c_\mathrm{u} &= 2Z_0 Z_\mathrm{c}(\cos^2\theta - \cos\theta_+\cos\theta_-),\\
c_2 &= -2Z_0 Z_\mathrm{c}\cos\theta(\cos\theta_+ - \cos\theta_-),\\
c_3 &= (Z_\mathrm{c}^2 - Z_0^2)\cos\theta(\cos\theta_+ + \cos\theta_-),\\
\Delta &= (Z_\mathrm{c}^2 + Z_0^2)\cos\theta(\cos\theta_+ + \cos\theta_-) + 2Z_0 Z_\mathrm{c}(\cos^2\theta + \cos\theta_+\cos\theta_-),
\end{aligned} \tag{2}$$

where $Z_0 = \sqrt{\mu_0/\varepsilon_0}$ and $Z_\mathrm{c} = \sqrt{\mu/(\varepsilon + \mu\xi^2)}$ are the wave impedances of vacuum and the chiral medium, respectively. We introduce the unit matrix I and the Pauli matrices [21]:

$$\sigma_2 = \begin{bmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}, \quad \sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}. \tag{3}$$

The reflection matrix M_R can be rewritten as

$$M_\mathrm{R} = c_\mathrm{u} I + c_\varphi \sigma_\varphi, \tag{4}$$

where $c_\varphi = \sqrt{c_2^2 + c_3^2}$, $\sigma_\varphi = \sigma_2 \sin\varphi + \sigma_3\cos\varphi$, $\sin\varphi = c_2/c_\varphi$, and $\cos\varphi = c_3/c_\varphi$.

The no reflection condition is satisfied when M_R has a zero eigenvalue, namely, $\det(M_\mathrm{R}) = 0$ or $\operatorname{rank}(M_\mathrm{R}) \leq 1$. For the incident wave with the corresponding eigenpolarization, the reflection is nullified. From Eq. (4), we observe that the eigenvalue problem for M_R reduces to that for σ_φ. The eigenvalues of σ_φ are ± 1 and their corresponding eigenpolarizations are $\boldsymbol{e}_{\varphi+} = \cos(\varphi/2)\boldsymbol{e}_z + \mathrm{i}\sin(\varphi/2)(\boldsymbol{e}_x\sin\theta + \boldsymbol{e}_y\cos\theta)$ and $\boldsymbol{e}_{\varphi-} = \sin(\varphi/2)\boldsymbol{e}_z - \mathrm{i}\cos(\varphi/2)(\boldsymbol{e}_x\sin\theta + \boldsymbol{e}_y\cos\theta)$, where $\boldsymbol{e}_x$, $\boldsymbol{e}_y$, and $\boldsymbol{e}_z$ are the unit vectors in the direction of the positive x-, y-, and z-axes, respectively. Therefore, M_R has a zero eigenvalue when $c_\mathrm{u} = c_\varphi$ ($c_\mathrm{u} = -c_\varphi$) is satisfied, and no reflection is achieved for the incident wave with the polarization $\boldsymbol{e}_{\varphi-}$ ($\boldsymbol{e}_{\varphi+}$).

Achiral case ($\xi = 0$)—The reflection matrix is $M_\mathrm{R} = c_\mathrm{u} I + c_3\sigma_3$. The eigenpolarizations are $\boldsymbol{e}_x\sin\theta + \boldsymbol{e}_y\cos\theta$ and $\boldsymbol{e}_z$; therefore, the no reflection condition can be satisfied only for linearly polarized (LP) waves. The no reflection effect is observed at a particular incident angle θ that satisfies $c_\mathrm{u} = \pm c_3$. The condition $c_\mathrm{u} = c_3$ ($c_\mathrm{u} = -c_3$) yields a no-reflection angle, called the Brewster angle, for TM (TE) waves in isotropic achiral media.

Chiral case ($\xi \neq 0$)—First, we consider a general case of $Z_\mathrm{c} \neq Z_0$, which gives $\varphi \neq n\pi/2$ with an integer n. The eigenpolarizations are $\boldsymbol{e}_{\varphi\pm}$; hence, the no reflection condition can be

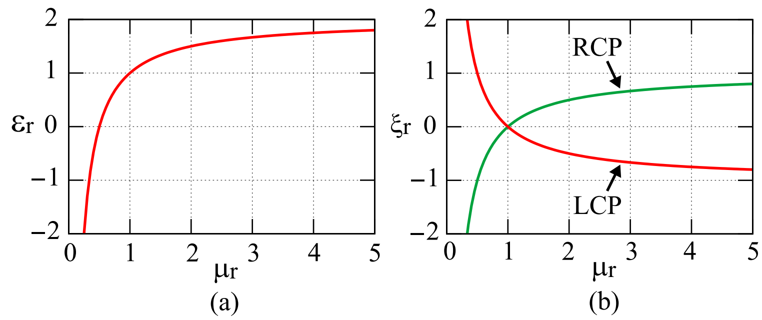

Fig. 2. (a) Relation between μ_r and ε_r and (b) between μ_r and ξ_r for no reflection conditions (invisible conditions). In the (μ_r-ξ_r) graph, the red and green lines represent the conditions for LCP and RCP waves, respectively.

satisfied only for elliptically polarized (EP) waves. The no reflection effect is observed at a particular incident angle θ satisfying $c_u = \pm c_\varphi$, which is a natural extension of the usually observed no reflection effect, or the Brewster effect in achiral media.

When $Z_c = Z_0$, we have $M_R = c_u I + c_2 \sigma_2$. The eigenpolarizations are $[\boldsymbol{e}_z \pm i(\boldsymbol{e}_x \sin\theta + \boldsymbol{e}_y \cos\theta)]/\sqrt{2}$; hence, the no reflection condition can be satisfied only for CP waves. The condition $\theta_+ = \theta$ ($\theta_- = \theta$) is required in order to satisfy $c_u = -c_2$ ($c_u = c_2$), which is the no reflection condition for LCP (RCP) waves. We note that once $k_+ = k_0$ ($k_- = k_0$) is satisfied by selecting the constants of medium, $c_u = -c_2$ ($c_u = c_2$) is satisfied for any θ. Namely, the no reflection condition is satisfied for all angles of incidence. This observation is quite different from the no reflection conditions for TM and TE waves in isotropic achiral media and for EP waves in isotropic chiral media. A qualitatively new mode of no reflection is obtained for LCP (RCP) waves in isotropic chiral media when the wave impedance matching condition $Z_c = Z_0$ and the wavenumber matching condition $k_+ = k_0$ ($k_- = k_0$) are satisfied simultaneously.

We derive the explicit relations among ε, μ, and ξ for the no reflection condition for CP waves. From the above discussion, both $Z_c = Z_0$ and $k_+ = k_0$ ($k_- = k_0$) are necessary and yield the following relations:

$$\varepsilon_r = 2 - \frac{1}{\mu_r}, \quad \xi_r = \mp\left(1 - \frac{1}{\mu_r}\right), \tag{5}$$

where $\varepsilon_r = \varepsilon/\varepsilon_0$, $\mu_r = \mu/\mu_0$ and $\xi_r = Z_0\xi$. The negative (positive) sign in Eq. (5) indicates the condition for LCP (RCP) waves. Figure 2(a) represents the (μ_r-ε_r) relation shown in Eq. (5), and Fig. 2(b) shows the (μ_r-ξ_r) relation. It should be noted that the no reflection condition can be satisfied by shifting the parameters $(\varepsilon_r, \mu_r, \xi_r)$ away from the vacuum parameters $(1, 1, 0)$ by a small amount. Such a medium can be obtained by using the state-of-art technology of metamaterials.

It is already known that under certain conditions, the no reflection effect for LP waves is observed at any incident angle in *anisotropic* achiral media [6]. In this case, the incident wave is refracted in the anisotropic medium, while for the present case, the incident wave is transmitted straight through the chiral medium. Thus, the medium with $Z_c = Z_0$ and $k_+ = k_0$ ($k_- = k_0$) can be considered as vacuum, namely, the medium is invisible, for LCP (RCP) waves.

3. Application of invisible medium for CP waves

We propose a CPBS as one of the applications of the invisible medium for CP waves. Here, we set the parameters of the chiral medium as $\varepsilon_r = 0.75$, $\mu_r = 0.8$, and $\xi_r = 0.25$, which give

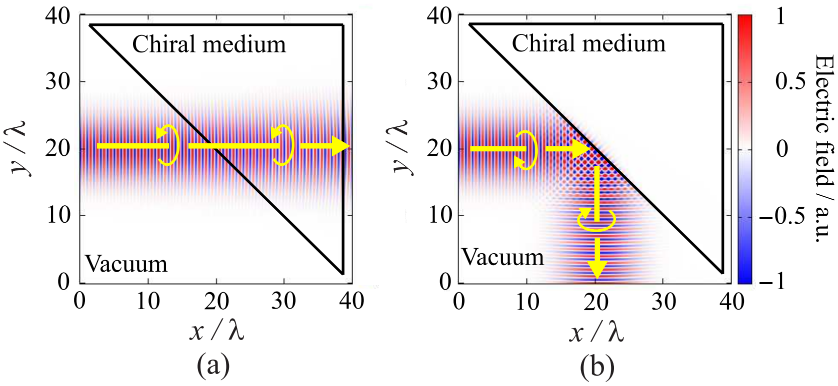

Fig. 3. Results of two-dimensional FDTD analysis of the CPBS. Propagation of (a) LCP waves and (b) RCP waves. Straight and curved arrows represent the propagation direction and polarization direction, respectively. λ is the wavelength of the EM waves.

the invisible condition for LCP waves. For $k_- = 0.6k_0$, Snell's equation for RCP waves is expressed as $\sin\theta = 0.6\sin\theta_-$; hence, the critical angle for RCP waves is $\theta_c = \arcsin(0.6) \simeq 37^\circ$. Therefore, LCP waves are completely transmitted without any reflection, while RCP waves are totally reflected with the incident angle greater than 37°. This implies that we can divide the incident waves into LCP and RCP waves.

We carry out an FDTD analysis [22] of the CPBS. It is assumed that the EM wave transmitted from vacuum is incident at an angle of 45° ($> \theta_c$) on a triangular prism made of the chiral medium. In order to adopt the two-dimensional FDTD method, Maxwell's equations for CP waves are rearranged as follows:

$$\frac{\partial E_z}{\partial y} = \mathrm{i}\omega(\mu \pm \mu\xi Z_c)H_x, \tag{6}$$

$$-\frac{\partial E_z}{\partial x} = \mathrm{i}\omega(\mu \pm \mu\xi Z_c)H_y, \tag{7}$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = -\mathrm{i}\omega\left[(\varepsilon + \mu\xi^2) \pm \frac{\mu\xi}{Z_c}\right]E_z, \tag{8}$$

where the relation $\boldsymbol{H} = \pm(\mathrm{i}/Z_c)\boldsymbol{E}$ [16] is used; and the positive (negative) sign corresponds to LCP (RCP) waves. Figure 3 shows the results (a) for LCP waves and (b) for RCP waves. It is observed that the LCP wave is transmitted straight through the chiral medium without any reflection and that the RCP wave is totally reflected at the surface of the chiral medium. It is confirmed that the incident wave can be split into LCP and RCP waves and circular polarizing beam splitter is achieved.

The advantages of the CPBS are as follows. A wide acceptance angle is obtained (53° in the above mentioned example). An incident wave with arbitrary polarization is distinctly split into LCP and RCP wave components with no losses. Anti-reflection coating is not required. A single element, i.e., one chiral prism, is sufficient for the CPBS. Although frequency sensitivity depends on material dispersion, broadband metamaterials make the realization of a frequency insensitive CPBS possible. The ideal properties of the CPBS make it an efficient polarizing beam splitter.

4. Physical meaning of invisible condition for CP waves

We consider the physical meaning of the invisible condition for CP waves in the chiral medium. For simplicity, let us assume that the invisible condition is satisfied for LCP waves.

First, we consider the medium polarization $\boldsymbol{P}$ and magnetization $\boldsymbol{M}$ induced by $\boldsymbol{E}$ and $\boldsymbol{B}$ in LCP waves. They are given by $\boldsymbol{P} = \boldsymbol{P}_{\mathrm{E}} + \boldsymbol{P}_{\mathrm{B}}$ and $\boldsymbol{M} = \boldsymbol{M}_{\mathrm{B}} + \boldsymbol{M}_{\mathrm{E}}$, where $\boldsymbol{P}_{\mathrm{E}} = (\varepsilon - \varepsilon_0)\boldsymbol{E}$, $\boldsymbol{P}_{\mathrm{B}} = -\mathrm{i}\xi\boldsymbol{B}$, $\boldsymbol{M}_{\mathrm{B}} = -(\mu^{-1} - \mu_0^{-1})\boldsymbol{B}$, and $\boldsymbol{M}_{\mathrm{E}} = \mathrm{i}\xi\boldsymbol{E}$ [28]. Taking into account the relation $\boldsymbol{H} = (\mathrm{i}/Z_{\mathrm{c}})\boldsymbol{E}$ that is satisfied for LCP waves [16], from Eqs. (1) and (5), it is not difficult to confirm that $\boldsymbol{P} = 0$ and $\boldsymbol{M} = 0$ are satisfied irrespective of the propagation direction. Owing to the electromagnetic mixing attributed to ξ, the polarization $\boldsymbol{P}_{\mathrm{B}}$ induced by the magnetic flux density completely cancels out the polarization $\boldsymbol{P}_{\mathrm{E}}$ induced by the electric field. Similarly, $\boldsymbol{M}_{\mathrm{E}}$ cancels out $\boldsymbol{M}_{\mathrm{B}}$. As a result of the destructive interference of electric and magnetic responses, net polarization and magnetization vanish in the case of LCP waves in the chiral medium; namely, the chiral medium is identical to vacuum for LCP waves. Therefore, for any angle of incidence, LCP waves are transmitted without reflection or refraction.

5. Summary and discussion

We studied the no reflection condition for chiral media. In addition to the no reflection effect for EP waves, which is a natural extension of the usual Brewster effect for LP waves, we found a qualitatively new mode of the no reflection effect. When the wave impedance matching condition $Z_{\mathrm{c}} = Z_0$ and the wavenumber matching condition $k_+ = k_0$ ($k_- = k_0$) were satisfied simultaneously, LCP (RCP) waves were transmitted from vacuum to an isotropic chiral medium without any reflection or refraction irrespective of the incident angle. The chiral medium was invisible for LCP (RCP) waves.

In this paper, we ignore losses of the media for simplicity. According to our estimation, the loss of metamaterials available today is small enough for the demonstration of no reflection phenomena. In fact, we have examined the TE Brewster condition using split ring resonators (SRRs) and obtained substantial reduction of reflection, $-27\,\mathrm{dB}$, successfully at near resonance frequency [12]. Metamaterials for chiral media can be designed by slightly modifying the structure of the SRRs [31], so it will be possible to experimentally demonstrate the no reflection effect for circularly polarized wave.

We proposed a CPBS as a straightforward application of the invisible medium for CP waves. The CPBS was a simple structure, i.e., one prism made of the chiral medium, and the ideal properties of CPBS make it an efficient polarizing beam splitter. We believe that the invisible medium for CP waves can be used in many other applications. For example, we could fabricate invisible containers for CP waves. The container can physically confine fluids or gases; however, it is invisible for CP waves. Another example is circular-polarization-selective waveguides that transmit only one of the CP waves for which the invisible condition is *not* satisfied.

For future studies, it is necessary to prepare metamaterials whose ε_{r}, μ_{r}, and ξ_{r} satisfy the invisible condition for CP waves. Such metamaterials can be realized by employing chiral structures [31] in the microwave region and by electromagnetically induced chirality in atomic systems [32, 33] in the optical region.

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